



# COMMON PRE-BOARD EXAMINATION: 2022-23

Class-XII Subject: MATHEMATICS (041)



Date: 12/01/2023

Time: 3 hours

M.M.: 80

## MARKING SCHEME

	SECTION -A(MCQ)	M
1.	(b) $\pi /6$	1
2.	(d) 5	1
3.	(a) 16	1
4.	(d) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	1
5.	(a) $\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$	1
6.	(b) 1	1
7.	(d) $\frac{33}{7}$	1
8.	b) 2	1
9.	(a) 1	1
10.	(b) 0	1
11.	(a) $-\tan(1-x) + C$	1
12.	(b) $1+x^2$	1
13.	(b) 4	1
14.	(b) -3	1

<b>15.</b>	(c) $\langle 2/3, 2/3, 1/3 \rangle$	<b>1</b>
<b>16.</b>	(a) $A \subseteq B$	<b>1</b>
<b>17.</b>	(d) at every point of the line-segment joining the points $(0.6, 1.6)$ and $(3, 0)$	<b>1</b>
<b>18.</b>	(c) 0.28	<b>1</b>
<b>19.</b>	(d) A is false but R is true.	<b>1</b>
<b>20.</b>	(c) A is true but R is false.	<b>1</b>
SECTION-B		
<b>21.</b>	a)=1 b) $\pi/6$  OR  One-one Onto	<b>2</b>
<b>22.</b>	Let at any time 't' length of edge of cube is x and its volume is V, then $V=x^3$ .....(i) and this edge is increasing at the rate of 3 cm/s. $\therefore dx/dt=3$ cm/s. We have to find, rate of change of volume V When $x=10$ cm $\therefore dv/dt=3x^2dx/dt$ $\Rightarrow dv/dt=3x^2 \cdot 3=9x^2$ When $x=10$ cm, $dv/dt=9(10)^2=900$ cm <sup>3</sup> /s Hence, Rate of change of Volume is 900cm <sup>3</sup> /sec.	<b>2</b>
<b>23.</b>	$\begin{aligned}(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) &= \mathbf{a} \times \mathbf{a} - \mathbf{b} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} - \mathbf{b} \times \mathbf{b} \\&= \mathbf{0} - [ -(\mathbf{a} \times \mathbf{b}) ] + \mathbf{a} \times \mathbf{b} - \mathbf{0} \\&= \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{b} \\&= 2(\mathbf{a} \times \mathbf{b})\end{aligned}$	<b>2</b>

**24.**

We know that projection of  $\vec{a}$  on  $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$\Rightarrow 4 = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \quad \dots(i)$$

$$\text{Now, } \vec{a} \cdot \vec{b} = 2\lambda + 6 + 12 = 2\lambda + 18$$

$$\text{Also } |\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{4 + 36 + 9} = 7$$

Putting in (i) we get

$$4 = \frac{2\lambda + 18}{7}$$

$$\Rightarrow 2\lambda = 28 - 18 \Rightarrow \lambda = \frac{10}{2} = 5$$

**OR**

The vector and cartesian equations of the line passing through the point (5,2,4) and parallel to the vector  $3i+2j-8k$  are

$$r = 5i + 2j + 4k + \lambda(3i + 2j - 8k) \text{ and } \frac{x-5}{3} = \frac{y-2}{2} = \frac{z-4}{-8}.$$

**2****25.**

$$x = a(1 - \cos\theta), y = b(\theta - \sin\theta)$$

Differentiating x and y w.r.t. x, we get

$$\frac{dx}{d\theta} = a \frac{d}{d\theta}(1 - \cos\theta)$$

$$= a[0 - (-\sin\theta)] = a \sin\theta$$

and

$$\frac{dy}{d\theta} = b \frac{d}{d\theta}(\theta - \sin\theta)$$

$$= b(1 - \cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{b(1 - \cos\theta)}{a \sin\theta}$$

$$= \frac{b \times \sin^2\left(\frac{\theta}{2}\right)}{a \times 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)}$$

$$= \left(\frac{b}{a}\right) \tan\left(\frac{\theta}{2}\right).$$

**2****SECTION-C****26.**

$$8 + 3x - x^2 \text{ can be written as } 8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$$

$$\text{Therefore } 8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) = \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$$

$$I = \int \frac{1}{\sqrt{8 + 3x - x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx$$

$$\text{Let } x - \frac{3}{2} = t \Rightarrow dx = dt \Rightarrow I = \int \frac{1}{\sqrt{\frac{41}{4} - t^2}} dt = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{\sqrt{41}}\right) + C = \sin^{-1}\left(\frac{x - \frac{3}{2}}{\sqrt{41}}\right) + C = \sin^{-1}\left(\frac{2x - 3}{\sqrt{41}}\right) + C$$

**3**

27.

Let  $E_1$  and  $E_2$  be two events such that

$E_1$  = A coming to the school in time.

$E_2$  = B coming to the school in time.

Here  $P(E_1) = 3/7$  and  $P(E_2) = 5/7$

$\Rightarrow P(\bar{E}_1) = 4/7$ ,  $P(\bar{E}_2) = 2/7$

$P(\text{only one of them coming to the school in time}) = P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$

$$= 3/7 \times 2/7 + 5/7 \times 4/7 = (6 + 20)/49 = 26/49$$

3

### OR

(i) Sum of probabilities = 1

$$\text{i.e., } 0+k+2k+2k+3k+2k^2+7k^2+k=1 \\ 10k^2+9k=1 \quad \text{or} \quad 10k^2+9k-1=0$$

$$(k+1)(10k-1)=0, \quad k=-1 \quad \text{or} \quad k=\frac{1}{10}$$

$$k \neq -1 \quad \therefore \quad k=\frac{1}{10}$$

$\therefore$  The probability distribution is

X	0	1	2	3	4	5	6	7
P(X)	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$

$$(ii) \quad P(X < 3) = P(0) + P(1) + P(2) = 0$$

$$+ \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$$

$$(iii) \quad P(X > 6) = P(7) = \frac{7}{100} + \frac{1}{10} = \frac{7+10}{100} = \frac{17}{100}$$

28.

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we get

$$A+C=0$$

$$A+B-2C=1$$

$$-2A+2B+C=0$$

On solving these equations, we get

$$A=\frac{2}{9}, B=\frac{1}{3} \text{ and } C=-\frac{2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x+1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9} \int \frac{1}{(x+1)} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{(x+2)} dx$$

$$= \frac{2}{9} \log|x+1| + \frac{1}{3} \left( \frac{-1}{x-1} \right) - \frac{2}{9} \log|x+2| + C$$

$$= \frac{2}{9} \log \left| \frac{x+1}{x+2} \right| - \frac{1}{3(x-1)} + C$$

3

29.

$$(1+y^2)dx = (\tan^{-1}y - x)dy$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

Hence

$$IF = e^{\int \frac{1}{1+y^2} dy}$$

$$= e^{\tan^{-1}y}$$

Hence the above differential equation changes to

$$e^{\tan^{-1}y} \cdot \frac{dx}{dy} + \frac{xe^{\tan^{-1}y}}{1+y^2} = \frac{e^{\tan^{-1}y} \tan^{-1}y}{1+y^2}$$

$$e^{\tan^{-1}y} \cdot dx + \frac{xe^{\tan^{-1}y}}{1+y^2} dy = \frac{e^{\tan^{-1}y} \tan^{-1}y}{1+y^2} dy$$

$$d(e^{\tan^{-1}y} \cdot x) = d(e^{\tan^{-1}y})$$

Integrating both sides give us

$$e^{\tan^{-1}y} \cdot x = e^{\tan^{-1}y} + C$$

## OR

$$\text{Given differential equation } x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$$

$$\frac{dy}{dx} = \frac{y \cos y/x + x}{x \cos y/x} \dots\dots (i)$$

It is homogenous differential equation,

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$(i) \Rightarrow v + x \frac{dv}{dx} = \frac{v x \cos v + x}{x \cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + 1}{\cos v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + 1 - v \cos v}{\cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1}{\cos v}$$

$$\Rightarrow \cos v dv = \frac{dx}{x}$$

Integrating both sides

$$\Rightarrow \sin v = \log|x| + C$$

$$\Rightarrow \sin \frac{y}{x} = \log |x| + C$$

30.

$$\text{Corner point } Z = 3x + 9y$$

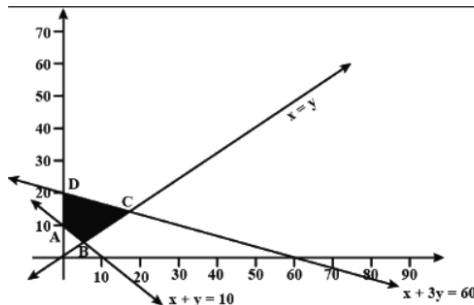
$$A = (0,10) \text{ 90}$$

$$B = (5,5) \text{ 60}$$

$$C = (15,15) \text{ 180}$$

$$D = (0,20) \text{ 180}$$

From the graph maximum value of X occurs at two corner points C(15,5) and D(0,20) with value 180 and minimum occurs at point B(5,5) with value 60.



**Maximise at all the point lying on the line CD**

3

31.

$$\text{Let } I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx \dots \quad (1)$$

$$I = \int_0^\pi \left\{ \frac{(\pi - x) \tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} \right\} dx, \quad \left( \because \int_0^a f(x)dx = \int_0^a f(a-x)dx \right)$$

$$\Rightarrow I = \int_0^\pi \left\{ \frac{-(\pi - x) \tan x}{-(\sec x + \tan x)} \right\} dx$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x) \tan x}{\sec x + \tan x} dx \dots \quad (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^\pi \frac{\pi \tan x}{\sec x + \tan x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\sin x + 1 - 1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi 1 \cdot dx - \pi \int_0^\pi \frac{1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi[x]_0^\pi - \pi \int_0^\pi \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi^2 - \pi \int_0^\pi (\sec^2 x - \tan x \sec x) dx$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan x - \sec x]_0^\pi$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan \pi - \sec \pi - \tan 0 + \sec 0]$$

$$\Rightarrow 2I = \pi^2 - \pi[0 - (-1) - 0 + 1]$$

$$\Rightarrow 2I = \pi^2 - 2\pi$$

$$\Rightarrow 2I = \pi(\pi - 2)$$

$$\Rightarrow I = \frac{\pi}{2}(\pi - 2)$$

**OR**

$$I = \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$I = \int_0^{\frac{\pi}{2}} \left( \sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}} \right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \left( \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} \right) dx$$

$$\text{Let } z = \sin x - \cos x, dz = (\cos x + \sin x) dx$$

$$z^2 = \sin^2 x + \cos^2 x - 2 \sin x \cos x$$

$$I = \int_{-1}^1 \frac{\sqrt{2}}{\sqrt{1-t^2}} dz$$

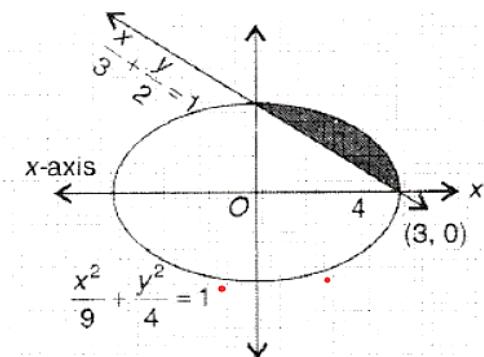
$$I = [\sqrt{2} \sin^{-1} t]_{-1}^1$$

$$I = \pi \sqrt{2}$$

### SECTION D

**32.**

**5**



Area of shaded region

$$= \int_0^3 \left\{ \frac{2}{3} \sqrt{9-x^2} - \frac{2}{3}(3-x) \right\} dx$$

$$= \frac{2}{3} \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} + \frac{(3-x)^2}{2} \right]_0^3$$

$$= \frac{2}{3} \left[ \left( 0 + \frac{9\pi}{2} + 0 \right) - \left( 0 + 0 + \frac{9}{2} \right) \right]$$

$$= \frac{2}{3} \left( 9 \frac{\pi}{4} - \frac{9}{2} \right)$$

$$= 3 \left( \frac{\pi}{2} - 1 \right) \text{ sq. unit.}$$

33.

5

R is relation defined on  $N \times N$  by  $(a,b) R (c,d)$  if and only if  $ad = bc$ , then R is

**(i) Reflexive** Since, for any  $a, b \in N$

$$\Rightarrow ab = ba \Rightarrow (a,b) R (a,b)$$

**(ii) Symmetric** Let  $(a,b) R (c,d)$

$$\Rightarrow ad = bc \Rightarrow bc = ad$$

$$\Rightarrow cd = da \quad (\because \text{commutativity of natural numbers})$$

$$\Rightarrow (c,d) R (a,b)$$

**(iii) Transitive** Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$$\Rightarrow ad = bc \text{ and } cf = de$$

$$\Rightarrow ade = bce \Rightarrow acf = bce$$

$$\Rightarrow af = be \quad (\text{cancellation law})$$

$$\Rightarrow (a,b) R (e,f)$$

Hence, from above observations, we conclude that R is an equivalence relation on  $N \times N$

**OR**

For a relation to be reflexive  $xRx$

For real x

$$xRx \Rightarrow x - x + \sqrt{2} = \sqrt{2}$$

$\sqrt{2}$  is an irrational number.

$\therefore xRx$  is reflexive.

For a relation to be symmetric  $xRy = yRx$

For real number x and y

$$xRy \Rightarrow x - y + \sqrt{2}$$

$$yRx \Rightarrow y - x + \sqrt{2}$$

$$\Rightarrow xRy \neq yRx$$

$\therefore$  The relation is not symmetric.

	<p>For a relation to be transitive <math>xRy = yRz \Rightarrow xRz</math></p> <p>For real numbers x, y and z</p> <p>Let</p> $\begin{aligned} x &= -\sqrt{2} \\ y &= 3\sqrt{2} \\ z &= 2 \end{aligned}$ <p>[Substitute the values of x, y, and z in relation]</p> $\begin{aligned} xRy \Rightarrow x - y + \sqrt{2} &= -\sqrt{2} - 3\sqrt{2} + \sqrt{2} \\ &= -3\sqrt{2} \text{ is an irrational number.} \end{aligned}$ $\begin{aligned} yRz \Rightarrow y - z + \sqrt{2} &= 3\sqrt{2} - 2 + \sqrt{2} \\ &= 4\sqrt{2} - 2 \text{ is an irrational number} \end{aligned}$ $\begin{aligned} xRz \Rightarrow x - z + \sqrt{2} &= -\sqrt{2} - 2 + \sqrt{2} \\ &= -2 \text{ is not an irrational number} \end{aligned}$ <p><math>\therefore xRy, yRx</math> then x is not related to z The relation is not transitive.</p>	
34.	<p>This can be written as <math>\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \dots\dots\dots(1)</math></p> <p><math>\therefore</math> the coordinates of any point on the line is</p> $x = 4 - 2\lambda, y = 6\lambda, z = 1 - 3\lambda$ <p>Let Q(4-2λ, 6λ, 1-3λ) be the foot of perpendicular from the point P (2, 3, -8) on line (1)</p> <p>We know the direction ratios of any line segment PQ is given by</p> $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ <p>The direction cosines of PQ is given by</p> $= (-2\lambda + 4 - 2, 6\lambda - 3, -3\lambda + 1 + 8)$ $= (-2\lambda + 2, 6\lambda - 3, -3\lambda + 9)$ <p>Now Q is the foot of the perpendicular of the line (1)</p> <p><math>\vec{PQ}</math> is the perpendicular to the line (1)</p> <p>hence the sum of the product of this direction ratios is 0</p> $= (-2\lambda + 2)(-2) + (6\lambda - 3).6 + (-3\lambda + 9)(-3) = 0$ $\Rightarrow 4\lambda - 4 + 36\lambda - 18 + 9\lambda - 27 = 0$ $\Rightarrow 49\lambda - 49 = 0$ $\therefore \lambda = 1$ <p>Substituting <math>\lambda = 1</math> in Q we get</p> <p>So that Q(2, 6, -2) and distance of PQ = <math>3\sqrt{5}</math> units</p>	5

**OR**

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\text{or } \vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$$

$$\text{with } \vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \quad \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k},$$

and comparing line

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

$$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\text{with } \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \text{ and } \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$\therefore$  Distance between

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \text{ and } \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| \dots (1)$$

$$\therefore \vec{a}_2 - \vec{a}_1 = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= 0 + \hat{j} - 4\hat{k} = \hat{j} - 4\hat{k}$$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = (-\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= (-2+4)\hat{i} - (2+2)\hat{j} + (-2-1)\hat{k}$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{171} = \sqrt{29}$$

$$\therefore d = \left| \frac{(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right|$$

$$= \left| \frac{-4 + 12}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}$$

35.

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix} = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9) = 67 \neq 0$$

5

For  $\text{adj } A$  :

$$A_{11} = -6 \quad A_{21} = 17 \quad A_{31} = 13$$

$$A_{12} = 14 \quad A_{22} = 5 \quad A_{32} = -8$$

$$A_{13} = -15 \quad A_{23} = 9 \quad A_{33} = -1$$

$$\therefore \text{adj. } A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj. } A$$

$$= \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

The given system of equation can be written as

$$AX = B \Rightarrow X = A^{-1}B \dots \text{(i)}$$

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

Putting the value of  $X$ ,  $A^{-1}$  and  $B$  in (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = 3, y = -2, z = 1$$

SECTION E		
36.	<p>i) A polynomial is everywhere differentiable in its domain,</p> <p>ii) <math>f(x) = 3x^4 + 4x^3 - 12x^2 + 12</math></p> <p>Differentiating with respect to <math>x</math>, we get</p> $f'(x) = 12x^3 + 12x^2 - 24x$ $= 12x(x^2 + x - 2)$ $= 12x(x - 1)(x + 2)$ <p>Now, by putting <math>f'(x) = 0</math>, we get <math>x = -2, 0, 1</math>.</p> <p><b>Critical points=-2,0 and 1</b></p> <p>iii) <math>f(x)</math> is Increasing in <math>(-2, 0)</math> and in <math>(1, \infty)</math>.</p> <p>so in given interval it is increasing in <math>(-2, 0)</math> and <math>(1, 3)</math></p> <p style="text-align: center;">OR</p> <p>iii) max abs=255 at <math>x=3</math> and min abs=-20 at <math>x=-2</math></p>	<b>1+1+</b> <b>2</b>
37.	<p>P(<math>x_1, y_1</math>) is on the curve <math>y = x^2 + 7 \Rightarrow y_1 = x_1^2 + 7</math></p> <p>Distance from P(<math>x_1, x_1^2 + 7</math>) and (3, 7)</p> $D = \sqrt{(x_1 - 3)^2 + (x_1^2 + 7 - 7)^2}$ $\Rightarrow \sqrt{(x_1 - 3)^2 + (x_1^2)^2}$ $\Rightarrow D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$ $D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$ $D' = x_1^4 + x_1^2 - 6x_1 + 9$ $\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6 = 0$ $\frac{d^2D'}{dx^2} = 12x_1^2 + 2$ $\left. \frac{d^2D'}{dx^2} \right _{x_1=1} = 12 + 2 = 14 > 0$ <p><math>x_1 = 1</math> and <math>2x_1^2 + 2x_1 + 3 = 0</math> gives no real roots</p> <p>The critical point is (1, 8).</p> $\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6$ $\left. \frac{d^2D'}{dx^2} \right _{x_1=1} = 12 + 2 = 14 > 0$ <p>Hence distance is minimum at (1, 8).</p> $D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$ $D = \sqrt{1 + 1 - 6 + 9} = \sqrt{5} \text{ units}$	<b>1+1+</b> <b>2</b>

38.	<p>Let <math>E_1</math> be the event that person has a disease, <math>E_2</math> be the event that person does not have a disease and <math>A</math> be the event that blood test is positive.</p> <p>As <math>E_1</math> and <math>E_2</math> are the events which are complementary to each other.</p> <p>Then <math>P(E_1) + P(E_2) = 1</math></p> <p><math>\Rightarrow P(E_2) = 1 - P(E_1)</math></p> <p>Then <math>P(E_1) = 0.1\% = 0.1/100 = 0.001</math> and <math>P(E_2) = 1 - 0.001 = 0.999</math></p> <p>Also <math>P(A E_1) = P(\text{result is positive given that person has disease}) = 99\% = 0.99</math></p> <p>And <math>P(A E_2) = P(\text{result is positive given that person has no disease}) = 0.5\% = 0.005</math></p> <p>Now the probability that person has a disease, given that his test result is positive is <math>P(E_1 A)</math>.</p> <p><b>So total probability = <math>P(E_1).P(A E_1) + P(E_2).P(A E_2)</math></b></p> $= \frac{1}{1000} \times \frac{99}{100} + \frac{999}{1000} \times \frac{5}{1000} = \frac{990}{1000000} + \frac{4995}{1000000} = \frac{5985}{1000000} = 0.005985 = \frac{1197}{2000}$ <p>By using Bayes' theorem, we have</p> $P(E_1 A) = \frac{P(E_1).P(A E_1)}{P(E_1).P(A E_1) + P(E_2).P(A E_2)}$ <p>Substituting the values we get</p> $= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$ $= \frac{0.00099}{0.00099 + 0.004995}$ $= \frac{0.00099}{0.005985} = \frac{990}{5985} = \frac{110}{665}$ $\Rightarrow P(E_1 A) = \frac{22}{133}$	2+2